Journal of Transport and Supply Chain Management
ISSN: (Online) 1995-5235, (Print) 2310-8789

Using fuzzy expectation-based programming for inventory management

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Dates:
Received: 05 Apr. 2022
Accepted: 25 Aug. 2022
Published: 28 Sept. 2022

How to cite this article:
Widowati, W., Sutrisno, S. & Tjahjana, R.H., 2022, 'Using fuzzy expectation-based programming for inventory management', *Journal of Transport and Supply Chain Management* 16(0), a782. https://doi.org/10.4102/jtscm.v16i0.782

Background: Order allocation planning and inventory management are two important problems in manufacturing industries that must be solved optimally to gain maximal profit. Commonly, there are several unknown parameters in those problems such as future price, future demand, etc., and this means decision-making support that can handle this uncertainty is needed to calculate an optimal decision.

Objectives: This study aimed to propose a newly developed joint decision-making support to solve order allocation planning and inventory optimisation of raw materials in a production system comprising multiple suppliers, products and review times with fuzzy parameters.

Method: The model was formulated as a fuzzy expectation-based quadratic programming with the uncertain parameters approached as fuzzy numbers. This was used to handle the fuzzy parameters involved in the problem. A classical optimisation algorithm, the generalised reduced gradient combined with branch-and-bound embedded in LINGO 18.0 was applied to calculate the optimal decision. Numerical experiments were conducted using some randomly generated data with four suppliers, four raw materials and six review times.

Results: Results provided the optimal decision for the given problem, that is, the number of raw materials to be ordered from each supplier at each review time, as well as the corresponding number to be stored in the warehouse.

Conclusion: The proposed model successfully solved the given problems and thus can be used by decision-makers to solve their order allocation planning and inventory problems.

Keywords: joint decision-making support; fuzzy parameters; fuzzy programming; inventory optimisation; order allocation problem.

Introduction

Manufacturing corporations need decision-making support to determine the best decision required to solve their problems, and this includes the use of several approaches such as the mathematical optimisation model. This further involved the application of classical models for problems with certain parameters, while there is a need to develop the appropriate support for those with uncertain parameters such as future price, demand, transportation cost, etc. This study was therefore conducted to fill this gap by focusing on the raw material order allocation problem and inventory optimisation.

In general, problems with different specifications need different tools used to solve the problem. Some papers have been published on the development of decision-making support for order allocation planning problems and inventory optimisation under various situations and specifications. It was discovered that most of them employed mathematical optimisation model approaches to calculate the optimal decision by optimising some objective functions such as operational cost. Meanwhile, each model is specially designed for specific problems with corresponding characteristics. For example, a linear programming model was developed to solve order allocation problems with deterministic parameters (Kara 2011; Ware, Singh & Banwet 2014); a linear integer programming approach was employed to solve order allocation problems with price discounts (Adi Wicaksono et al. 2018); an optimisation approach was used for supplier selection with disruption risks (Hosseini et al. 2019); a mathematical model was used for order allocation problems with a quantity discount and fast service options (Alegoz & Yapicioglu 2019); and a bi-objective optimisation was used to manage risks on order allocation problems (Vishnu, Das & Sridharan 2019). Moreover, several case study researches have also applied formulated decision-making support in different fields such as logistics services (Hu et al. 2018), the rubber industry (Sembiring, Matondang & Dalimunthe 2019), cement production (Ismail & Mahardika 2017), the power source industry (Firoz, Biswal & Satapathy 2018), the textile industry (Nazar et al. 2019) and retail shops (Tandel et al. 2020). These previously mentioned supports are applicable for deterministic cases with the values of all parameters in the problem known. Some other models dealing with unknown parameters have also been applied such as probabilistic theory and fuzzy theory. For example, an order allocation problem, which considered a full truckload with unknown demand, was effectively solved by probabilistic optimisation model (Sutrisno & Wicaksono 2017), while a multi-objective optimisation approach focused on uncertain scoring was used (Khoshfetrat, Rahiminezhad Galankashi & Almasi 2019) to solve order allocation under risk and inflation conditions.

It is therefore possible to solve problems containing unknown parameters using uncertainty theory. The most basic type is the probabilistic theory, where the unknown value is treated as a random variable with some corresponding probability distribution function formulated using the observation data to ensure suitability. However, in a situation where there is no fund to collect data, it is possible to use the fuzzy theory with the unknown parameter treated as a fuzzy variable and the probability distribution function replaced by fuzzy membership function introduced in Zadeh (1978). This further leads to the development of fuzzy programming to solve an optimisation containing fuzzy parameters (Liu & Liu 2002). This has been reported to be a proper decision-making tool to solve many problems such as data envelopment analysis (Ghasemi et al. 2015; Shiraz, Charles & Jalalzadeh 2014; Wang & Chin 2011), grinding optimisation (Li 2014; Virivinti & Mitra 2014), portfolio management (Gupta, Mittal & Mehlawat 2013; Moussa, Kamdem & Terraza 2014; Wang, Li & Watada 2017), health optimisation (Mezei & Nikou 2018), groundwater source management (Milan, Roozbahani & Banihabib 2018), vehicle parking management (Faddel, Al-Awami & Abido 2017), energy portfolio optimisation (Kouaissah & Hocine 2020), batch production systems (Javid et al. 2020) and several others.

The literature review above showed that the optimisation-based decision-making tool for order allocation planning integrated with inventory optimisation in a fuzzy uncertainty environment is limited. A number of models have been proposed in dealing with integrating order allocation planning and inventory control; however, each model has its own environment and suits the considered environment. For example, an optimisation model was proposed in Qu et al. (2022) for the problem with multi-echelon distribution network; however, all parameters were known with certainty and therefore do not work in an uncertain environment.

In this article, a newly developed decision-making tool is proposed in the form of fuzzy quadratic programming to produce a joint optimal decision for integrated order allocation problems and inventory optimisation containing fuzzy parameters. The problem discussed involved multiple suppliers, multiple raw materials and multiple review times. The proposed model aims to calculate the optimal decision under the fuzzy uncertainty condition, that is, to calculate the number of each raw material to be ordered from each supplier at each review time such that the expectation of the total operational cost for the whole time horizon is minimal. In order to show the decision-making process, some numerical examples are discussed in the numerical experiment section.

Methodology

Problem definition

Consider a manufacturing or a retail company that needs to procure some raw materials or products from several suppliers in different future review time instants (e.g. daily) under uncertainty conditions where there are a number of uncertain parameters. The flow of the raw materials or products considered in this problem is illustrated in Figure 1. Three parties are involved, namely suppliers, warehouse and production units or buyers. The raw materials or products are procured from suppliers, stored in the warehouse and then used for production or sold to buyers. Each supplier has its own performance in terms of capacities, prices, defect rates, shortage rates and transport costs. The uncertain parameters are treated as fuzzy variables. Under this fuzzy uncertainty, the decision-maker wants to calculate the number of each raw materials to be ordered from each supplier at each review time instant and those to be stored in the inventory to minimise the operational costs such as those associated with purchasing, transportation, the penalty for damaged product and holding. To deal with this problem, some assumptions were made as follows:

1. The raw material will not expire up to the end of the review time instant optimisation.
2. The shortage of raw material from the supplier will be delivered at the following review time instant.

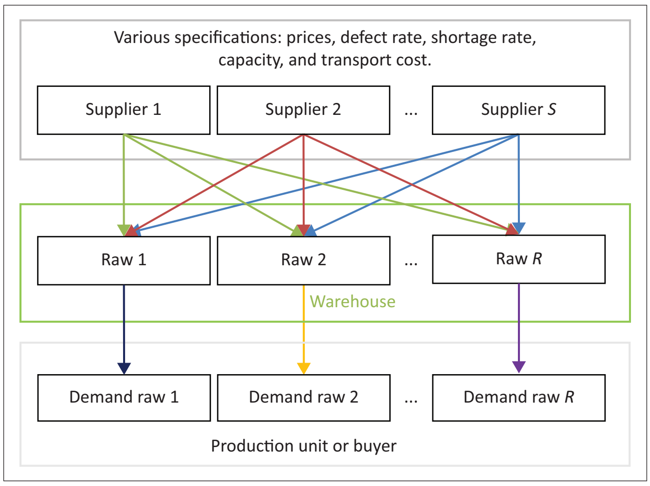

FIGURE 1: The supply chain composed of suppliers, warehouse and production unit or buyer.

3. The transportation mechanism involves using different trucks for different suppliers, and this means transport cost is recorded for each supplier.
4. There are four uncertain parameters concerning raw material prices, demands, defect rates and shortage rates. This means that values for these parameters are not known a priori and could change at any time. The decision is then calculated from the expectation of their fuzzy membership functions. The rest of the parameters is assumed to be known a priori.
5. The distance of suppliers to the warehouse are vary; this is covered by the transport cost for each supplier. However, it is assumed that the raw materials are arrived at the warehouse within the same review time as they are purchased.

The decision-maker, under the assumptions above, seeks the allocation of purchasing raw materials to suppliers and the quantity of that stored in the inventory such that the inventory level follows a reference point as closely as possible, and the expectation of the total operational cost for the whole time planning horizon is minimal; this is the problem considered in this study. Furthermore, the methodology employed in this study is summarised as problem-solving steps listed as follows. Firstly, the problem is defined and the assumptions are stated. This step also includes identifying the fuzzy parameters. Then, the objective function to be optimised is modelled, that is, the expectation of the total operational cost. A term for controlling the inventory level is added in the objective function as a quadratic function of the difference between the actual inventory level and its reference point. Minimising this term brings the actual inventory level to its reference point. Next, the constraint functions are modelled based on the situations that should be satisfied as defined in the problem definition. After that, the fuzzy expectation value–based programming is employed to calculate the optimal decision. This was chosen as it can handle optimisation problems containing fuzzy parameters (see Liu 2009) for technical notes about fuzzy programming. Finally, the derived optimal decision is implemented or executed.

Mathematical notations

The notations used in the mathematical optimisation model include the following indices:

k : review time instant

s : supplier

r : raw material

i : possible outcome.

Decision variables:

X_{ksr} : the number of raw material r ordered from supplier s at a review time instant k

Y_{ks} : a binary decision variable (1 if some raw materials are ordered from supplier s at a review time instant k, 0 if otherwise)

X_{kr} : a recourse decision variable representing the shortage of raw material r at review time instant k if the available raw material is not satisfying the demand

I_{kr} : the number of raw materials r stored in the warehouse/inventory at review time instant k to be used for the following review time instant.

Fuzzy parameters:

UP_{ksr} : a fuzzy variable declaring the price per unit of raw materials r ordered from supplier s at a review time instant k

DE_{kr} : a fuzzy variable declaring the number of raw materials r needed (demanded) for production at a review time instant k

DR_{ksr} : a fuzzy variable declaring the percentage of defected raw materials r because of damage or transport loss from the supplier s at a review time instant k

SR_{ksr} : a fuzzy variable declaring the shortage (in percentage) of raw materials r ordered from supplier s at a review time instant k.

Crisp parameters:

TC_{ks} : transportation cost for supplier s at a review time instant k

DC_{ksr} : unit penalty cost for defected raw material r from the supplier s at a review time instant k

SC_{ksr} : unit penalty cost for shortage raw material r from the supplier s at a review time instant k

RC_{kr} : recourse cost to procure one unit raw material r needed to satisfy the demand at a review time instant k

MC_{ksr} : the maximum number of raw materials r that can be supplied by supplier s at a review time instant k

HC_{kr} : holding cost per unit per time instant of raw material r to be stored in the warehouse at a review time instant k

MW_{kr} : the maximum number of raw materials r that can be stored in the warehouse at a review time instant k

$\mathrm{I}_{kr}^{(ref)}$: reference point for inventory level

IC_{kr} : inventory reference point tracking cost (or weight) or raw material r at a review time instant k.

Other notations:

ξ : a fuzzy variable

μ_ξ : membership function of the fuzzy variable ξ

$\hat{E}[\xi]$: fuzzy expectation value of the fuzzy variable ξ.

Mathematical model

In the proposed model, the decision variables were calculated and executed before the unknown parameters occur. This means the number of raw materials needed for production may not be satisfied by the available raw material in the warehouse and by the current procurement at any review time instant. Therefore, a decision-maker needs alternative decisions by ignoring the shortage or buying some other amount of each raw material denoted by X_{kr} with some recourse cost per unit already defined by RC_{kr} with the assumption the recourse raw material will be available instantly at the current review time instant.

The objective function to be minimised is the total operational cost consisting of the following components:

1. The expectation of purchasing cost for all raw materials bought from all suppliers at all review time instants:

$$Z_1 = \hat{E}\left[\sum_{k=1}^{K}\sum_{s=1}^{S}\sum_{r=1}^{R} X_{ksr} \cdot UP_{ksr}\right]. \quad \text{[Eqn 1]}$$

2. The expectation of transportation cost to transport all raw materials bought from all suppliers at all review time instants:

$$Z_2 = \sum_{k=1}^{K}\sum_{s=1}^{S} TC_{ks} \cdot Y_{ks}. \quad \text{[Eqn 2]}$$

3. The expectation of the cost of losing some raw materials because of lesser quality or damage during the delivery process:

$$Z_3 = \hat{E}\left[\sum_{s=1}^{S}\sum_{p=1}^{P} DR_{kr} \cdot DC_{ksr} \cdot X_{ksr}\right]. \quad \text{[Eqn 3]}$$

4. The expectation of cost for raw material shortage:

$$Z_4 = \hat{E}\left[\sum_{k=1}^{K}\sum_{s=1}^{S}\sum_{p=1}^{P} SR_{kr} \cdot SC_{ksr} \cdot X_{ksr}\right]. \quad \text{[Eqn 4]}$$

5. Holding or inventory cost for storing raw materials in the warehouse at all review time instants:

$$Z_5 = \sum_{k=1}^{K}\sum_{r=1}^{R} HC_{kr} \cdot \mathrm{I}_{kr}, \quad \text{[Eqn 5]}$$

6. The trajectory or reference point tracking cost for inventory level in the quadratic form:

$$Z_6 = \sum_{k=1}^{K}\sum_{r=1}^{R}\left[IC_{kr} \cdot \left(\mathrm{I}_{kr} - \mathrm{I}_{kr}^{(ref)}\right)^2\right]. \quad \text{[Eqn 6]}$$

This also describes that the decision to store raw material in the inventory is made as close as possible to the reference point $\mathrm{I}_{kr}^{(ref)}$, which is treated as a 'safe' or 'reserve' point by the decision-maker.

7. The expectation of recourse cost for satisfying the demand (if any):

$$Z_7 = \hat{E}\left[\sum_{k=1}^{K}\sum_{r=1}^{R} X_{kr} \cdot RC_{kr}\right]. \quad \text{[Eqn 7]}$$

The complete form of the proposed mathematical optimisation model is written, that is, the objective function together with the constraint functions in the following where the explanation of each constraint function will follow:

$$\min Z = Z_1 + Z_2 + Z_3 + Z_4 + Z_5 + Z_6 + Z_7 \quad \text{[Eqn 8]}$$

subject to:

$$\begin{aligned}&\mathrm{I}_{(k-1)r} + \hat{E}\left[\begin{aligned}&\sum_{s=1}^{S} SR_{(k-1)sr} \cdot X_{(k-1)sr} + \sum_{s=1}^{S} X_{ksr}\\ &-\sum_{s=1}^{S} DR_{ksr} \cdot X_{ksr} - \sum_{s=1}^{S} SR_{ksr} \cdot X_{ksr}\end{aligned}\right]\\ &\quad -\mathrm{I}_{kr} + X_{kr} \geq \hat{E}\left[DE_{kr}\right],\\ &\quad \forall r = 1,2...,R, \quad \forall k = 1,2...,K;\end{aligned} \quad \text{[Eqn 9]}$$

$$Y_{ks} = \begin{cases} 1, \text{ if} \sum_{r=1}^{R} X_{ks} > 0 \\ 0, \text{ otherwise}\end{cases}, \forall s = 1,2,...,S, \quad \forall k = 1,2,...,K; \quad \text{[Eqn 10]}$$

$$X_{ksr} \leq MS_{ksr}, \forall k = 1,2,...,k, \quad \forall s = 1,2,...,S, \forall r = 1,2,...,R; \quad \text{[Eqn 11]}$$

$$\mathrm{I}_{kr} \leq MW_{kr}, \forall k = 1,2,...,k, \quad \forall r = 1,2,...,R; \quad \text{[Eqn 12]}$$

$$\begin{aligned}&X_{ksr}, X_{kr}, I_{kr} \geq 0 \text{ and integer}, Y_{ks} \in \{0,1\}, \ \forall k = 1,2,..., K,\\ &\forall s = 1,2,...,S, \ \forall r = 1,2,...,R.\end{aligned} \quad \text{[Eqn 13]}$$

The constraint functions to be satisfied are explained as follows:

1. For review time instant $k = 1,2,3,...,K$, the expectation of raw materials stored in the warehouse from the previous review time instant plus those from the shortage of the previous review time instant plus those ordered at current review time instant minus the number of defected ones at current review time instant minus the shortage at current review time instant minus those to be stored at current review time instant plus recourse decision of the raw material number at current review time instant is expected to be bigger than the demand expected at current review time instant, which is formulated as (2), where $\mathrm{I}_{(0)r}$ $SR_{(0)sr}$, and $X_{(0)sr}$ represent the initial inventory, which is the available raw materials available on the inventory, initial shortage rate and the raw material ordering amount before the current review time instant, respectively. The unavailability of raw materials and purchasing before the current review time leads to the setting of these values to zero.
2. Assignment for binary decision variable Y_{ks} indicating the supplier s is selected to supply raw materials or not at the review time instant k (Y_{ks}= 1 if some raw materials are purchased to the supplier s, 0 otherwise), which was modelled as the constraint function (3).
3. The number of raw materials ordered from each supplier is upper bounded by the supplier's capacity to supply the raw material at each review time instant, which was represented by the inequality (4).
4. The number of raw materials stored in the warehouse is upper bounded by the warehouse's maximum capacity at each review time instant, which was modelled as the inequality (5).
5. The assignment for decision variables to be non-negative and integer; this is described by the constraint (6).

This optimisation model belongs to uncertain programming as it contains fuzzy variables. This therefore means it can be solved using fuzzy expectation-based programming, and the solution is guaranteed as the feasible set is closed and bounded as long as it is not empty. This method was selected because, mainly, it can handle fuzzy parameters in the problem. Moreover, it works by converting the uncertain programming into deterministic programming by expanding the objective function and the constraint functions using the fuzzy expectation theory. The optimal decision is then calculated by exploiting optimisation algorithms for deterministic programming (see e.g. Liu 2009 for more technical procedures regarding this). The method used to solve this optimisation problem is explained in the following subsections.

Fuzzy expectation value

The fuzzy expectation–based programming contains a calculation of the fuzzy expectation value; this is explained as follows. Consider fuzzy variables ξ with corresponding membership function μ_ξ. The expectation value of ξ is denoted by $\hat{E}[\xi]$. Generally, the fuzzy membership function is continuous with the discrete type used in this study. Therefore, the membership function of the fuzzy variable ξ is defined in the form as follows:

$$\mu_\xi = \begin{cases} \mu_1, \ \xi = \xi^{(1)}; \\ \mu_2 \ \xi = \xi^{(2)}; \\ \vdots \\ \mu_n, \xi = \xi^{(n)}; \\ 0, \text{others} \end{cases} \quad \text{[Eqn 14]}$$

where $\mu_1 \in (0,1)$ and $\xi^{(i)} \in \mathbb{R}, i = 1,\ldots,n$ and $\xi^{(1)} < \xi^{(2)} < \ldots < \xi^{(n)}$. Then, the fuzzy expectation value of ξ is $\hat{E}[\xi] = \sum_{i=1}^{n} w_i \cdot \xi^{(i)}$ where:

$$\omega_i = \frac{1}{2}\left(\max_{1\le j\le i} \mu_j + \max_{i\le j\le n} \mu_j - \max_{1\le j\le i^-} \mu_j \max_{1\le j\le n} \mu_j \right) \quad \text{[Eqn 15]}$$

for $i=1,2,\ldots,n$ (Liu 2009). Meanwhile, other results focused on the linearity of fuzzy expectation value, that is, for any two independent fuzzy variables ξ_1 and ξ_2 and $a,b \in \mathbb{R}$, then $\hat{E}[a\xi_1 + b\xi_2] = a\hat{E}[\xi_1] + b\hat{E}[\xi_2]$.

Fuzzy expectation–based programming

Fuzzy expectation–based programming applied in this article deals with the minimisation of one quadratic objective function subject to some linear constraint functions. Let $\chi = (\chi_1, \chi_1 \ldots, \chi_n) \in \mathbb{R}^n$ be the vector of decision variables, $\xi_i, i = 1,2,\ldots, m$ be the fuzzy variable with membership function, $\mu_{\xi i}: \mathbb{R} \to \mathbb{R}$, $\xi = (\xi_1,\ldots,\xi m)$ be the vector of fuzzy variables, $f(\chi,\xi)$: $\mathbb{R}^{n+m} \to \mathbb{R}$ be the objective function to be minimised, and $g_i(\chi,\xi)$, $i=1,2,\ldots,p$ are constraint functions. The general form of the optimisation may be expressed as:

$$\{\min f(x,\xi) \text{ s.t } g_i(x,\xi) \ge 0, i = 1,2,\ldots, p.\} \quad \text{[Eqn 16]}$$

To solve this, it is converted to deterministic programming by taking the expectation value of the objective function and the constraint functions, that is (Liu 2009):

$$\min \hat{E}[f(x,\xi)] \text{ s.t } \hat{E}[g_i(x,\xi)] \ge 0, i = 1,2,\ldots, p. \quad \text{[Eqn 17]}$$

Results and discussions

Numerical experiment results

Considering the order allocation and inventory optimisation problem with two raw materials denoted by R1 and R2, three supplier alternatives are denoted by S1, S2 and S3, and six review time instants are denoted by K1, K2, K3, K4, K5 and K6, modelled as an optimisation problem with fuzzy variables UP_{ksr}, DE_{kr}, DR_{ksr} and SR_{ksr} for $k=1,\ldots,6, s = 1,2,3$, and $r = 1,2$. The numerical experiment for this problem was conducted to illustrate the proposed model's solution with the data used generated randomly. The crisp parameters TC_{ks}, DC_{ksr}, SC_{ksr}, RC_{kr}, MS_{ksr}, HC_{kr}, MW_{kr} and $I_{kr}^{(ref)}$ for $k=1,\ldots,6$, $s = 1,2,3$, and $r = 1,2$ are provided in Table 1, Table 2, Table 3, Table 4 and Table 5, while the membership function of the fuzzy variables for $k=1,\ldots,6, s = 1,2,3$, and $k=1,\ldots,6, s = 1,2,3$, and $r = 1,2$ and is given by:

$$\mu_{UP_{ksr}} = \begin{cases} \mu^{(i)}_{UP_{ksr}}, & UP_{ksr} = UP^{(i)}_{ksr}, i = 1,\ldots,5; \\ 0, & \text{others}; \end{cases} \quad \text{[Eqn 18]}$$

$$\mu_{DE_{kr}} = \begin{cases} \mu^{(i)}_{DE_{ksr}}, & DE_{kr} = DE^{(i)}_{kr}, i = 1,\ldots,5; \\ 0, & \text{others}; \end{cases}$$

$$\mu_{DR_{ksr}} = \begin{cases} \mu^{(i)}_{DR_{ksr}}, & DR_{ksr} = DR^{(i)}_{ksr}, i = 1,\ldots,5; \\ 0, & \text{others}; \end{cases}$$

$$\mu_{SR_{ksr}} = \begin{cases} \mu^{(i)}_{SR_{ksr}}, & SR_{ksr} = SR^{(i)}_{ksr}, i = 1,\ldots,5; \\ 0, & \text{others}; \end{cases}$$

where $\mu_\xi^{(i)}$, $i=1,\ldots,5$ and the corresponding weight value used to calculate the fuzzy expectation for all the fuzzy variables ξ, $w_{\xi^{(i)}}$, $i=1,\ldots,5$ are presented in Table 6, Table 7, Table 8 and Table 9. The reference point for the inventory level was five units each for raw materials R1 and R2. Therefore, the generalised reduced gradient algorithm was employed to solve this optimisation problem with the integer solution derived by employing the branch-and-bound algorithm. The calculation was performed using LINGO 18.0 (Lindo Systems, Inc., Chicago, Illinois, United States) optimisation software in a common personal computer with specifications 3.2 GHz dual-core processor and 8 GB memory.

TABLE 1: Transportation cost.

k	Supplier		
	S1	S2	S3
All	45	42	45

TABLE 2: Raw material defect cost.

k	Supplier	Raw material	
		R1	R2
All	S1	1.00	2.00
	S2	1.00	1.50
	S3	1.50	1.50

TABLE 3: Raw material shortage cost.

k	Supplier	Raw material	
		R1	R2
All	S1	1	2
	S2	1.5	2.5
	S3	1.5	2

TABLE 4: Maximum capacity of the supplier.

k	Supplier	Raw material	
		R1	R2
All	S1	20	30
	S2	20	35
	S3	25	20

TABLE 5: Value for some crisp parameters.

k	Parameter	Raw material	
		R1	R2
All	RC_{kr}	40	45
	HC_{kr}	2	3
	MW_{kr}	20	25
	$I_{kr}^{(ref)}$	5	5
	IC_{kr}	1	1

TABLE 6: Parameter value for $\mu_{UP_{ksr}}$.

k	i	$UP_{ksr}^{(i)}$	Membership value $\mu_{UP_{ksr}}^{(i)}$	$w_{UP_{ksr}^{(i)}}$
All	1	22	0.2	0.1
	2	23	0.6	0.2
	3	24	1	0.25
	4	25	0.9	0.25
	5	26	0.4	0.2

It is possible to solve the optimisation problem for $k = 1,...,6$ in a one-time calculation to achieve the optimal solution, but a very long computational time was required using the common computer previously described. Assuming the decision-maker has only computers with common specifications, because of the computational time limitation, the problem was solved for every two review time instants instead of the six in the one-time calculation, and this also required a very long computational time. Meanwhile, with the assumption that the decision-maker has a limited time to make decisions, computation of solution to was interrupted after approximately 3 h. Therefore, substituting all parameters' values for only two review time instants $k = 1,2$ produced 26 core decision variables and 43 constraints with 40 fuzzy variables and 625 different scenarios based on the realisation of all fuzzy variables.

TABLE 7: Parameter value for $\mu_{DE_{ksr}}$.

k	i	$DE_{ksr}^{(i)}$	Membership value $\mu_{DE\,ksr}^{(i)}$	$w_{DE_{ksr}^{(i)}}$
All	1	8	0.3	0.15
	2	10	0.9	0.3
	3	12	1	0.2
	4	14	0.7	0.1
	5	16	0.5	0.25

TABLE 8: Parameter value for $\mu_{DR_{ksr}}$.

k	i	$DR_{ksr}^{(i)}$	Membership value $\mu_{DR\,ksr}^{(i)}$	$w_{DR_{ksr}^{(i)}}$
All	1	0.01	0.4	0.2
	2	0.02	0.9	0.25
	3	0.04	1	0.15
	4	0.05	0.8	0.2
	5	0.08	0.4	0.2

TABLE 9: Parameter value for $\mu_{SR_{ksr}}$.

k	i	$SR_{ksr}^{(i)} SR_{ksr}^{(i)}$	Membership value $\mu_{SR\,ksr}^{(i)}$	$w_{SR_{ksr}^{(i)}}$
All	1	0.01	0.8	0.4
	2	0.02	1	0.15
	3	0.25	0.9	0.25
	4	0.03	0.4	0.05
	5	0.35	0.3	0.15

The probabilistic optimisation problem was converted into deterministic equivalent optimisation with 16 250 decision variables and 39 979 constraints, and the procedure was repeated for review time instants $k = 3,4$ and $k = 5,6$. The optimal decision for raw material procurement and inventory optimisation is shown in Figure 2 and Figure 3, respectively.

The review time instant 0 was treated as the initial condition or initial value, with the raw material ordering value and inventory level at the point used as the previous ordering value and previous inventory level for review time instant 1 to 2. After the calculation, the optimal decision was applied to review time instant $k = 1$. Therefore, Figure 2 shows 13 units of R1, and 12 units of R2 should be purchased from S2 and none from S1 and S3 for $k = 1$. Figure 3 shows the number of raw materials to be stored in the warehouse is 7 and 6 units for R1 and R2, respectively. These decisions were used as the initial value to calculate the new optimal decision with $k = 2,3$, and the results were used for review time instant $k = 2$ as shown in Figure 2 and Figure 3. The optimal decision was to order 13 units of R1 and 12 units of R2 from S2, while 5 units of R1 and 5 units of R2 should be stored in the inventory. The optimal decision for other review time instants 3–6 was computed and derived analogously. It is, however, only supplier S2 that was selected to supply raw materials. This shows that supplier S2 has the best performance over all specifications including the prices, defect rates and late delivery rates, among others. Nevertheless, with another parameter setting, multiple suppliers could be selected together with the corresponding optimal amount of each raw material type to be ordered. Meanwhile, review time instant 6 was treated as the last time of optimisation, and this means the optimal decision was derived while solving the optimisation for $k = 5,6$, and the

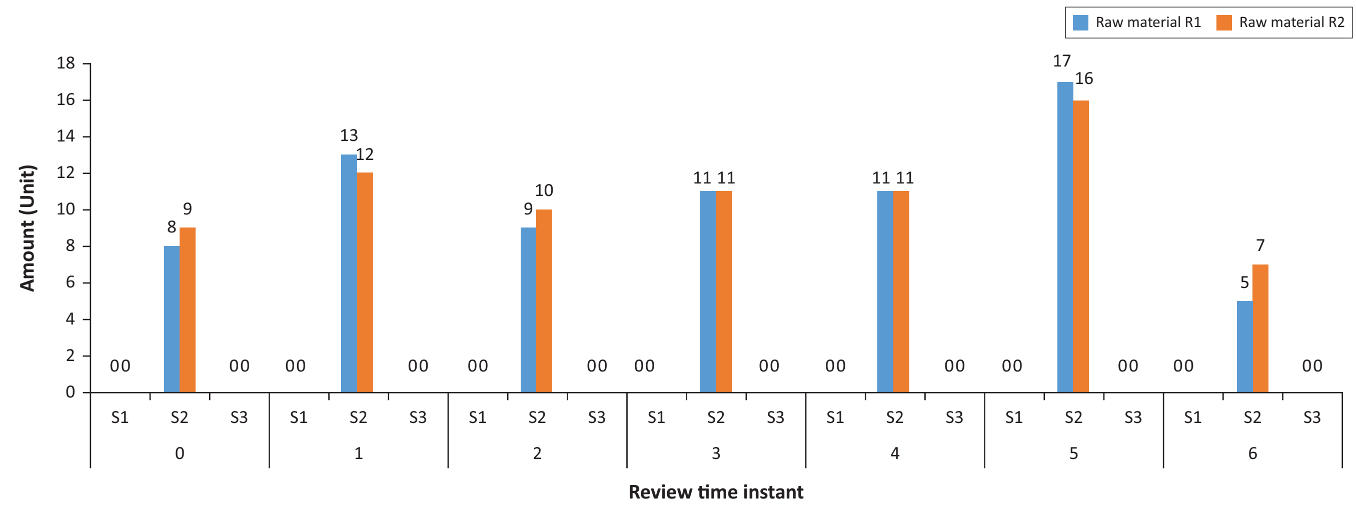

FIGURE 2: Optimal quantity of raw materials ordered from suppliers.

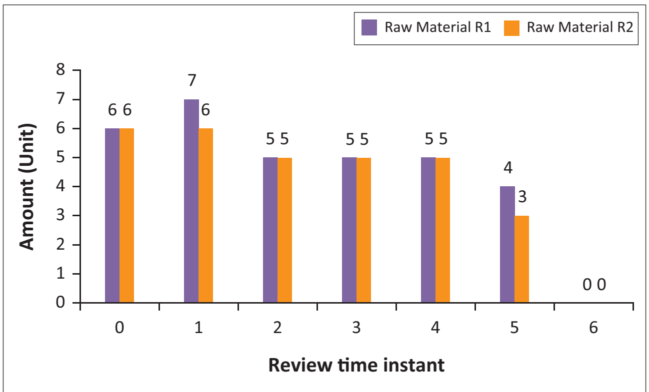

FIGURE 3: Inventory level.

results showed the inventory level should be zero. This was considered sensible because the manufacturer does not need raw materials anymore. Compared with some existing results, our proposed model is sensible as it was able to select the best supplier(s) and allocate the amount of the raw material to be ordered from the selected supplier(s). Similar results are shown in some reports (see e.g. Ahmad, Firouz & Mondal 2022; Amin-Tahmasbi et al. 2022; Lo et al. 2018; Qu et al. 2022; Sun, Guo & Li 2022; Wu, Gao & Barnes 2022).

Some observations regarding the computational aspects are discussed as follows. Figure 4a shows the objective value for each computation. It is, however, important to note the objective function value at each review time instant was computed for two review time instants. For example, at review time instant 1, the objective value was approximately 1150. The result was produced from optimising the model for review time instants 1 and 2, but the optimal decision was only for review time instant 1. This is the expectation value that means it is possible the real objective value is different after passing through the review time instant. Moreover, the value is higher with the demand as observed from 4 to 6. Figure 4b shows the objective value for the wait and see model, that is, the best possible outcome of all possible outcomes of the fuzzy parameters. It is much less than the expected objective value, meaning that the actual objective value could be much smaller than the expected value after all fuzzy parameters are revealed.

Meanwhile, Figure 4c shows the computational time applied to this experiment. It is important to note that the optimisation calculation was manually interrupted after some time because of the computational time limit as shown in Figure 4c. At review time instant 1, the computation was interrupted after about 2.8 h and later at approximately 8.4 h for 2 and so on. This scenario implies the derived decision is not the global optimiser but almost the optimal solution, and it was also observed to be sensible because of the assumption the decision-maker has no computational time anymore. Moreover, it was observed there was no significant change in the objective value (less than 5%) during and 1 h after the computation, based on the LINGO solver status. It was unfortunate that the graph of objective value versus computational time was not presented in this article because of the inability of the software to save the history for this objective value. Figure 4d shows the number of iterations of each computation at each review time instant, and it was discovered that a longer computational time leads to more iterations. These two concepts are linearly related, and even after more than 35 million iterations, the algorithm was still running, but the global optimal solution was not achieved. This shows the optimisation problem is complicated, huge and has a heavy load of computation.

Discussions and managerial insights

The computational experiment results showed that the proposed decision-making tool successfully solved the given problem. Therefore, this model can be adopted by decision-makers in industries to solve their supply chain problems. By implementing this model, both order allocation planning and inventory control problems are solved in an integrated way in one optimisation model; that is, the flow of raw materials is seen through the whole process. This approach is better than solving each of the order allocation planning and inventory optimisations independently. If the problem is solved independently, it is

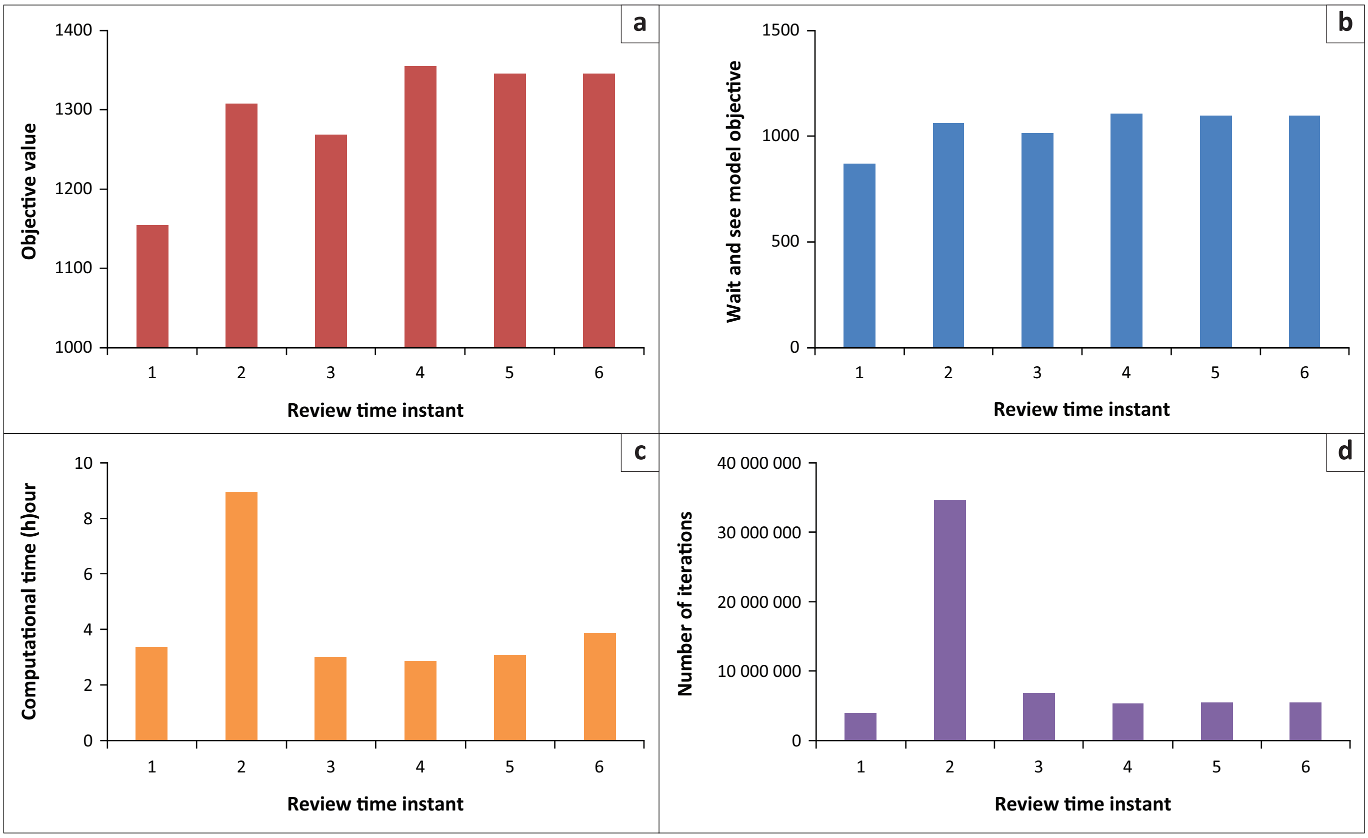

FIGURE 4: Computational aspects: (a) objective value at each review time instant, (b) wait and see model objective value, (c) computational time at each review time instant and (d) number of iterations of the computation at each review time instant.

possible to have unnecessary raw materials in between the two parties, as there is no synchronisation of the raw material flow, which could make a much higher operational cost. While using the proposed model, a further strategy related to the fuzzy membership function can be used by decision-makers to make a better decision. Decision-makers could utilise their intuition and experience in formulating the membership function; in general, the more accurate the membership function, the better the decision. Therefore, an experienced decision-maker could make a better decision.

Meanwhile, the inventory optimisation was considered as a reference point tracking problem, meaning that the decision-maker was assumed to be willing to store some number of raw materials in the warehouse. This suits decision-makers who want to have a safe inventory level and use the stored raw materials in unpredicted or unpredictable situations such as transportation disruption. Nevertheless, the reference point could be set as zero, meaning that the decision-maker wants to minimise the inventory level and does not want to have stock in the warehouse, which will provide a lower inventory cost. However, in this case, there is a bigger possibility for lower income when the on-hand raw materials do not satisfy the demand.

Furthermore, the results provide some managerial insights that are discussed as follows:

1. In principle, the proposed model can be implemented to order allocation and inventory management with any kind of raw materials or products provided that the assumptions and specifications are met. However, insignificant different specifications or assumptions are still possible to handle, such as the integer specification for the decision variables. One may consider that only some decision variables are integers when certain raw materials measurement are real numbers; in this case, the optimisation problem belongs to mixed-integer programming.
2. In practice, the proposed model can still be modified based on the specifications defined by the decision-maker; for instance, a fixed cost component can be added to the objective function.
3. If the values of some uncertain parameters are known a priori, these parameters can be treated immediately as crisp parameters. This will reduce the computational time.
4. Concerning the fuzzy membership functions, decision-makers may run the computation multiple times with different membership functions. The final decision can be derived from the results based on their experience and intuition; this is one of challenges in decision-making processes involving uncertain parameters without data.
5. A high-performance computer is more preferred to solve the optimisation problem, especially when it has a sufficiently large size. Assuming one review time instant means a day, the computation should be conducted before 24 h.

6. Decision-makers can add a computational time constraint to stop the computation after some time. This is usually needed for large-scale problems when the computational time takes longer time and the decision is already needed to be implemented.
7. The proposed model can be adjusted and modified based on the situations faced by the decision-maker. As an example, other cost components such as ordering cost to a supplier could be added; this can be done just by adding a new term in the objective function. Moreover, the constraint functions can also be modified based on the contract taken by both parties. For example, the first constraint function, in general, depends on the contract taken by both parties, whether the manufacturer or the supplier is responsible for the rejected or defective raw materials. In this case, the rejected raw materials are neglected, meaning that it is the manufacturer's responsibility. If it is the supplier's responsibility, a term could be added to the on-hand raw materials that comes from the defective raw materials at the previous review time instant, under the assumption that the rejected raw materials at the previous review time instant are replaced and arrived at the current review time instant.
8. The inventory reference point or trajectory tracking term in the objective function is used to bring the inventory level to its 'safe' point, which is decided by decision-makers. When decision-makers do not want to have this 'reserve' inventory, they could set the tracking cost as zero. In this case, the inventory level will be minimised so that the holding cost will be minimal.

Conclusion

Decision-making support via a mathematical optimisation model in the form of quadratic programming was considered in this article to determine the optimal decision for raw material procurement planning and inventory optimisation problem in a fuzzy environment. Some parameters in the model were approached as a fuzzy variable while the corresponding optimisation problem was solved in LINGO 18.0 optimisation software by employing generalised reduced gradient combined with branch-and-bound. Moreover, numerical experiments were conducted using three suppliers, two raw material types and six review time instants, and the optimal decision showed the quantity of raw material to purchase from each supplier and those to be stored in the inventory. The proposed decision-making tool can be used by decision-makers in manufacturing industries to solve their problems.

In the future, more complicated problems should be solved by developing the mathematical model by including the conditions attached to the problem, such as the addition of carrier service selection in the model. Furthermore, as the model tends to produce large-scale optimisation problems, it will be interesting to study the optimisation algorithms that are able to deal with the complexity. Some metaheuristic algorithms like a genetic algorithm and particle swarm optimisation are recommended to be studied in the near future.

Acknowledgements

Competing interests

The author declares that they have no financial or personal relationships that may have inappropriately influenced them in writing this article.

Authors' contributions

All authors contributed equally to this work.

Ethical considerations

This article followed all ethical standards for research without direct contact with human or animal subjects.

Funding information

The authors appreciate Universitas Diponegoro for research funding support under RPI Research Grant 2020 contract no. 474-89/UN7.6.1/PP/2020.

Data availability

Data sharing is not applicable to this article as no new data were created or analysed in this study.

Disclaimer

The views and opinions expressed in this article are those of the authors and do not necessarily reflect the official policy or position of any affiliated agency of the authors.

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